Hyper-lambda calculus for waitfree computation

Yoichi Hirai[1]

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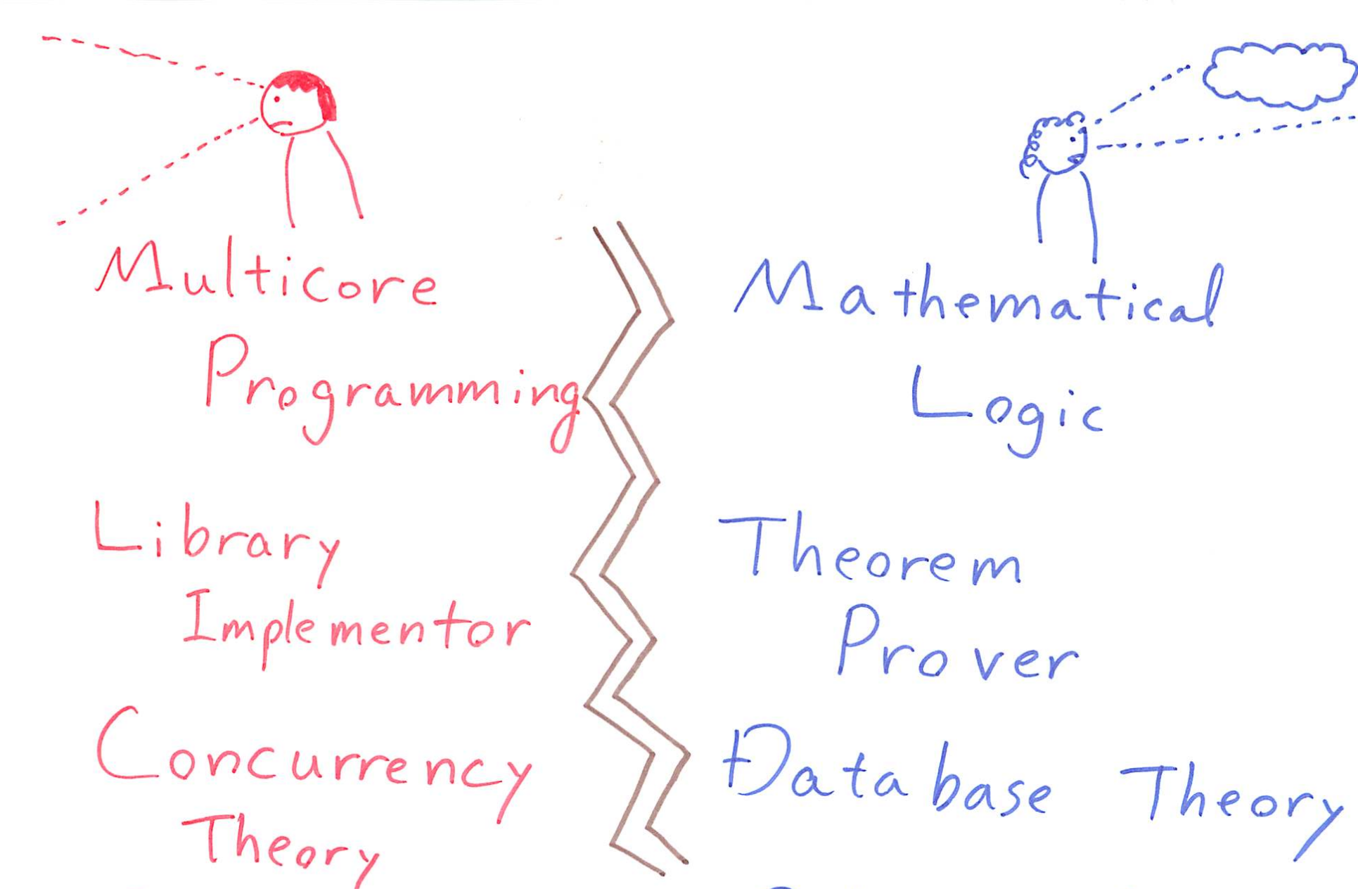
[1]Univ. of Tokyo, a JSPS fellow

Yoichi Hirai wants to be a traveling merchant



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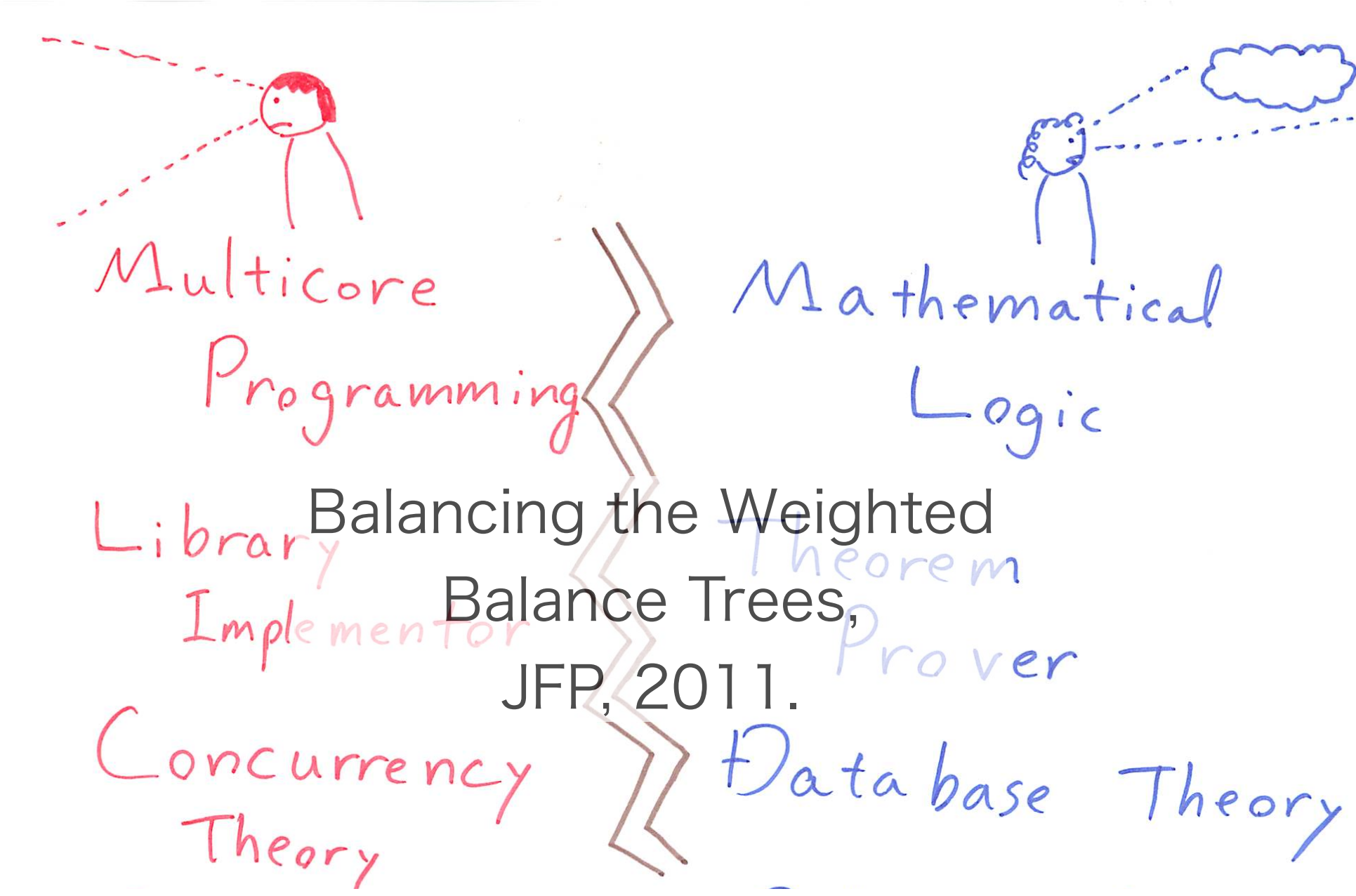

Gödel–Dummett logic

(nothing to do with completeness or incompleteness)

Your are familiar with Intuitionistic propositional logic

Add $(\varphi \supset \psi) \vee (\psi \supset \varphi)$ as provable.
(And anything provable with this in intuitionistic logic)

Gödel–Dummett logic

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Your are familiar with Intuitionistic propositional logic

Add $(\varphi \supset \psi) \vee (\psi \supset \varphi)$ as provable.
(And anything provable with this in intuitionistic logic)

Q. What chooses this disjunction in the corresponding lambda calculus?
An oracle does: $M : (\varphi \supset \psi) \vee (\psi \supset \varphi)$.
$M \rightsquigarrow^* \mathsf{inl}(\cdots)$? OR $M \rightsquigarrow^* \mathsf{inr}(\cdots)$?

Choosing an interesting oracle leads to an interesting application.
idea: formulas to inference rules!

Hypersequent formulation [Avron, 1991]

Hypersequent is a finite sequence of sequents.

Instead of the axioms $(\varphi \supset \psi) \vee (\psi \supset \varphi)$, include the communication rule

$$\frac{\mathcal{H}_0 \mid \Gamma_0, \Delta_0 \vdash \varphi_0 \qquad \mathcal{H}_1 \mid \Gamma_1, \Delta_1 \vdash \varphi_1}{\mathcal{H}_0 \mid \mathcal{H}_1 \mid \Gamma_0, \Delta_1 \vdash \varphi_0 \mid \Gamma_1, \Delta_0 \vdash \varphi_1}$$

- provable with cuts $\Longrightarrow$ provable without cuts (Gentzen's proof adopted)
- "it seems to us extremely important to determine the exact computational content of them [intermediate logics] — and to develop corresponding 'λ-calculi' "— Avron [1991].

Nobody has really done it

Hypersequent-style natural deduction [Baaz et al., 2001]

- normal form theorem proved via Avron's cut-elimination.
- reduction is not given

Parallel dialogue games [Fermüller, 2003]

- based on Lorenzen games.
- proof searching bottom-to-up with player knowing the answer proof tree and opponent directing for different parts of the proof.
- the global game state has local parts.
- for example, player can duplicate a local part into two (EC) 1
- already normalized proofs

Avron's question has not been answered.

Hypersequent-style natural deduction (modified Baaz et al. [2001])

$$\mathsf{Ax}\ \frac{}{\varphi \vdash \varphi}$$

$$\supset\mathcal{I}\ \frac{\mathcal{H} \mid \varphi, \Gamma \vdash \psi}{\mathcal{H} \mid \Gamma \vdash \varphi \supset \psi} \qquad \supset\mathcal{E}\ \frac{\mathcal{H} \mid \Gamma \vdash \varphi \supset \psi \qquad \mathcal{H}' \mid \Gamma \vdash \varphi}{\mathcal{H} \mid \mathcal{H}' \mid \Gamma \vdash \psi}$$

$\cdots$ (NJ rules) $\cdots$

$$\mathsf{com}\ \frac{\mathcal{H} \mid \varphi, \Gamma \vdash \psi \qquad \mathcal{H}' \mid \theta, \Delta \vdash \tau}{\mathcal{H} \mid \mathcal{H}' \mid \Gamma \vdash \theta \supset \psi \mid \Delta \vdash \varphi \supset \tau}$$

$$\mathsf{EC}\ \frac{\mathcal{H} \mid \Gamma \vdash \varphi \mid \Gamma \vdash \varphi}{\mathcal{H} \mid \Gamma \vdash \varphi} \qquad \mathsf{EW}\ \frac{\mathcal{H}}{\mathcal{H} \mid \Gamma \vdash \varphi}$$

and exchange rules.

$$\frac{}{x\colon \varphi \rhd x\colon \varphi}$$

$$\frac{\mathcal{G} \mid x\colon \varphi, \Gamma \rhd M\colon \psi}{\mathcal{G} \mid \Gamma \rhd \lambda x.M\colon \varphi \supset \psi}$$

$$\frac{\mathcal{G} \mid \Gamma \vdash M\colon \varphi \supset \psi \qquad \mathcal{G}' \mid \Gamma \vdash N\colon \varphi}{\mathcal{G} \mid \mathcal{G}' \mid \Gamma \vdash MN\colon \psi}$$

$\cdots$ (typed λ-cal. rules with $\mathcal{G} \mid$) $\cdots$

$$\frac{\mathcal{G} \mid x\colon \varphi, \Gamma \rhd M\colon \psi \qquad \mathcal{G}' \mid y\colon \theta, \Delta \rhd N\colon \tau}{\mathcal{G} \mid \mathcal{G}' \mid \Gamma \rhd \overrightarrow{l}\, x.M\colon \theta \supset \psi \mid \Delta \rhd \overleftarrow{l}\, y.N\colon \varphi \supset \tau}$$

(l: fresh location)

$$\frac{\mathcal{G} \mid \Gamma \rhd M\colon \varphi \mid \Gamma \rhd N\colon \varphi}{\mathcal{G} \mid \Gamma \rhd [M, N]\colon \varphi}$$

$$\frac{\mathcal{G}}{\mathcal{G} \mid \Gamma \rhd \mathsf{abort} : \varphi}$$

and exchange rules (internal and external).

Reduction: configuration $\rightsquigarrow$ configuration

Configuration is a triple $(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathcal{G})$
for $\overrightarrow{\sigma}, \overleftarrow{\sigma} : L \to \mathsf{Term} + \{\bot\}$ and a hyperterm $\mathcal{G}$.
We consider terms up to α-equivalence.

$$\frac{(\overrightarrow{\sigma}, \overleftarrow{\sigma}, M) \rightsquigarrow (\overrightarrow{\sigma}', \overleftarrow{\sigma}', O)}{(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathcal{G} \mid C[M] \mid \mathcal{G}') \rightsquigarrow (\overrightarrow{\sigma}', \overleftarrow{\sigma}', \mathcal{G} \mid C[O] \mid \mathcal{G}')}$$

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$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, (\lambda x.M)O) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, M[O/x])$ and other rules of the underlying λ-cal.

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for N closed:
$\left(\overrightarrow{\sigma}[I \mapsto O], \overleftarrow{\sigma}[I \mapsto \bot], (\overleftarrow{I}\, x.M)N\right) \rightsquigarrow (\overrightarrow{\sigma}[I \mapsto O], \overleftarrow{\sigma}[I \mapsto N], M[O/x])$

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conversion rules involving $[\quad , \quad]$

Reduction (contd.)

$$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \lambda x.\mathsf{abort}) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort})$$

$$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort}\, N) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort})$$

$$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, M\mathsf{abort}) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort})$$

$$\left(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \overrightarrow{l}\, x.\mathsf{abort}\right) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort})$$

$$\left(\overrightarrow{\sigma}, \overleftarrow{\sigma}, \overleftarrow{l}\, x.\mathsf{abort}\right) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, \mathsf{abort})$$

$$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, [\mathsf{abort}, M]) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, M)$$

$$(\overrightarrow{\sigma}, \overleftarrow{\sigma}, [M, \mathsf{abort}]) \rightsquigarrow (\overrightarrow{\sigma}, \overleftarrow{\sigma}, M)$$

Example Typing

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x : \mathsf{Nat} \rhd x : \mathsf{Nat}} \quad \dfrac{}{y : \mathsf{Rat} \rhd y : \mathsf{Rat}}}{\rhd \overrightarrow{I}\, x.x : \mathsf{Rat} \supset \mathsf{Nat} \;\mid\; \rhd \overleftarrow{I}\, y.y : \mathsf{Nat} \supset \mathsf{Rat}}}{\rhd (\overrightarrow{I}\, x.x)\,\text{  } : \mathsf{Nat} \;\mid\; \rhd (\overleftarrow{I}\, y.y)0 : \mathsf{Rat}}}{\rhd \mathsf{inl}\left((\overrightarrow{I}\, x.x)\,\text{  }\right) : \mathsf{Nat} \vee \mathsf{Rat} \;\mid\; \rhd \mathsf{inr}\left((\overleftarrow{I}\, y.y)0\right) : \mathsf{Nat} \vee \mathsf{Rat}}}{\rhd [\mathsf{inl}\left((\overrightarrow{I}\, x.x)\,\text{  }\right), \mathsf{inr}\left((\overleftarrow{I}\, y.y)0\right)] : \mathsf{Nat} \vee \mathsf{Rat}}$$

Example: reduction 1

$$
\begin{aligned}
&\left([], [], [\mathsf{inl}\left((\overrightarrow{\;\;}\, x.x)\right), \mathsf{inr}\left((\overleftarrow{\;\;}\, y.y)0\right)]\right) \\
\rightsquigarrow &\left([\;\;\mapsto], [], [\mathsf{inl}(\mathsf{abort}), \mathsf{inr}((\overleftarrow{\;\;}\, y.y)0)]\right) \\
\rightsquigarrow &\left([\;\;\mapsto], [], [\mathsf{abort}, \mathsf{inr}((\overleftarrow{\;\;}\, y.y)0)]\right) \\
\rightsquigarrow &\left([\;\;\mapsto], [], \mathsf{inr}((\overleftarrow{\;\;}\, y.y)0)\right) \\
\rightsquigarrow &\left([\;\;\mapsto], [\;\;\mapsto 0], \mathsf{inr}(\;\;)\right)
\end{aligned}
$$

Example: reduction 2

$$
\begin{aligned}
&\left([], [], [\mathsf{inl}\left((\overrightarrow{\ell}\, x.x)\,\text{🐭}\right), \mathsf{inr}\left((\overleftarrow{\ell}\, y.y)0\right)]\right) \\
\rightsquigarrow &\left([], [\ell \mapsto 0], [\mathsf{inl}\left((\overrightarrow{\ell}\, x.x)\,\text{🐭}\right), \mathsf{inr}\,(\mathsf{abort}\,)]\right) \\
\rightsquigarrow &\left([], [\ell \mapsto 0], [\mathsf{inl}\left((\overrightarrow{\ell}\, x.x)\,\text{🐭}\right), \mathsf{abort}\,]\right) \\
\rightsquigarrow &\left([], [\ell \mapsto 0], \mathsf{inl}\left((\overrightarrow{\ell}\, x.x)\,\text{🐭}\right)\right) \\
\rightsquigarrow &\left([\ell \mapsto \text{🐭}], [\ell \mapsto 0], \mathsf{inl}\,(0)\right)
\end{aligned}
$$

The schedule chooses left or right.

Theorems

Rewriting system:

1. strongly normalizable (simulation)
2. not confluent

Asynchrony:

1. Sound for waitfree computation (by SN)
2. not complete for waitfree computation
   - add an modality representing each thread?
   - change the logical connectives (seems harder)

Demo

In Haskell:

$$\text{com'}\ \frac{\dfrac{\triangleright\ \mathsf{getLine} : \mathsf{String}}{\triangleright\ \mathbf{b}_a\,(\mathsf{getLine}) : K_a\mathsf{String}} \quad \dfrac{\triangleright\ \mathsf{getLine} : \mathsf{String}}{\triangleright\ \mathbf{b}_b\,(\mathsf{getLine}) : K_b\mathsf{String}}}{K_a(\mathsf{String} \wedge \mathsf{String}) \mid K_b(\mathsf{String} \wedge \mathsf{String})}$$

Future work

(The traveling merchant shouts to)

- (computer scientists!)
  Compare with lambda-mu [Parigot, 1992] and symmetric lambda [Barbanera and Berardi, 1994] .
- (logicians!)
  classifying classical proofs on toopology of schedules (Saks and Zaharoglou [1993])
- (engineers!)
  Extend this logic (modal version) to higher-order to enable program extraction and verify some concurrent algorithms

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Michel Parigot. $\lambda\mu$-Calculus: An algorithmic interpretation of classical natural deduction. In *LNCS*, pages 190–201. Springer, 1992.

Franco Barbanera and Stefano Berardi. A symmetric lambda calculus for classical program extraction. In *Theoretical Aspects of Computer Software*, volume 789 of *LNCS*, pages 495–515. Springer, 1994.

Michael Saks and Fotios Zaharoglou. Wait-free k-set agreement is impossible: the topology of public knowledge. pages 101–110, 1993.